**RANDOMNESS:**
- Floating-Point Numbers & Issues
- Using Randomness
- (Pseudo) Random Numbers
- Monte Carlo Methods
- Percolation

**Scientific Numbers**

- $1.23664 \times 10^4$:
  - base: 10
  - exponent: 4
  - mantissa: 1.23664
  - precision: 6 decimals
  - sign

- $1.23664 \times 10^4 + 1.56333 \times 10^7$
  - shift-align the mantissa
  - do the subtraction
  - round to 6-digit precision
  - result: $1.56457 \times 10^4$
FP Arithmetic, IEEE 754

- Float: 1b sign + 8b exponent + 23b mantissa
- Double: 1b sign + 11b exponent + 52b mantissa
- 0.085 decimal = \[0 \ 01111011 \ 0101110001010001111011\] binary
- Decimal examples with 3-digit mantissa:
  - \[355 = 0.355 \times 10^3\]
  - \[12.77 = 0.128 \times 10^2\]
- Rounding by magnitude instead of directed, but ALU can do directed rounding needed for interval computing
  - round \[-0.3555\] ? \[-0.356\] or \[-0.355\]
Arithmetic Issues

• Precision = number of digits
  Accuracy = number of correct digits
  \( \pi = 3.133333 \) is precise to 7 decimals, but…

Clicker Question

Which of the following three is slowest?

A. Adding two 32bit integers
B. Fetching one 64bit double from RAM
C. Multiplying two 32bit floating point numbers
Arithmetic Issues

- Round-off error:
  \[ 0.124 \times 0.351 = 0.043524: 0.435 \times 10^{-1} \]
  \[ \Delta = 0.24 \times 10^{-4} \]
- Digit cancellation error:
  \[ 0.127 - 0.124 = 0.003: 0.300 \times 10^{-2} \]
- Large summations are problematic, as are iterated computations in general…

Examples

- Square root by Newton, role of epsilon
- Digit cancellation near zero for \((1 - \cos(x))/x^2\)
  - Ariane 5 rocket 1996: arithmetic overflow
  - Patriot missile 1991: time increment 1/10th sec
  - Intel FDIV hardware error 1997
Correct Graph?

Near zero for \((1-\cos(x))/x^2\) is what?

- \(\cos(x) \approx 1 - x^2/2 + x^4/24 - x^6/6! + \ldots\)
- \(1 - \cos(x) \approx x^2/2 - O(x^4)\)

So, \((1 - \cos(x))/x^2 \approx 1/2 - O(x^2)\)

We should see 0.5 at zero…

Which is **not** a Random Event?

A. Color of first car crossing Stadium Ave after 12 noon
B. 101st digit in expansion of \(\pi\) is even
C. Fair coin toss
D. June 1, 2008, is a cloudy day
Percolation

Probability for percolation given probability of generating the grid
Critical probability is 0.5

Percolation Grid is a Graph
Percolation Grid is a Graph

What Happens during Exploration?

• A recursive graph exploration:
  
  visit(graph_node X):
  mark X as “visited”
  for each adjacent, not visited Y, where X → Y:
    visit(Y)
  return

• This is a depth-first search
DFS of Graph

visit(graph_node X):
mark X as “visited”
for each adjacent, not visited Y, where X → Y:
visit(Y)
return

What Happens in Exploration?

• A recursive graph exploration, ending if we reach the other end:
  visit(graph_node X):
  mark X as “visited”
  if X at bottom, return “percolates”
  for each adjacent not visited Y where X → Y:
    s = visit(Y)
    if s is “percolates”: return s
    return “does not percolate”

• Driving by calling visit(X) with every X in top row