Topics for next two classes

- Introduction to recursion
- Recursive definitions
- Recursive functions
  - How to approach formulation and implementation

Reading: Zelle, Chapter 13.2

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What is recursion?

- Have you had someone tell you that you can't use a word in its own definition? This is a *circular* definition.
- A description of something that refers to itself, has a base case, and is defined on smaller instances is called a **recursive definition**.
- A function calling itself is called a **recursive function**
- Why use recursive functions?
  - Recursion allows simple definitions
  - Powerful programming technique

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Recursive definitions

- **Factorial**

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)! & \text{otherwise} \end{cases} \qquad n! = n(n-1)(n-2)\ldots(1)$$

  - fac(n) = n * fac(n-1)

- Recurrences like $t_1 = 5$; $t_n = 2*t_{n-1} + 5$
- Recursive definitions need a termination condition

More on recursion

- Recursive programming is directly related to mathematical induction, a common proof technique
  - Termination condition = base case
- Recursive definition are not circular because they work on smaller and smaller problem sizes and finally reach the base case
  - When the base case is encountered, the expression can be computed
- Functional programming languages (e.g., Scheme, Haskell, Lisp) use recursion for iteration

Clicker question

1. sum(0) = 1
   sum(n+1) = sum(n) + 1/(n+1)!

2. sum(0) = 0
   sum(1) = 3
   sum(n) = sum(n+1) + sum(n-2)

3. sum(0) = 1
   sum(n) = sum(n-2) + n^2

Which ones are incorrect recursive definitions?

A. 2
B. 2 and 3
C. 3
D. All are incorrect
E. All are correct

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Two functions computing n!

```
def non-rec-fact(n)
    fact = 1
    for factor in range(n,1,-1):
        fact = fact * factor
    return(fact)
```

```
def fact(n):
    if n==1:
        return 1
    else:
        return n*fact(n-1)
```

factorials.py

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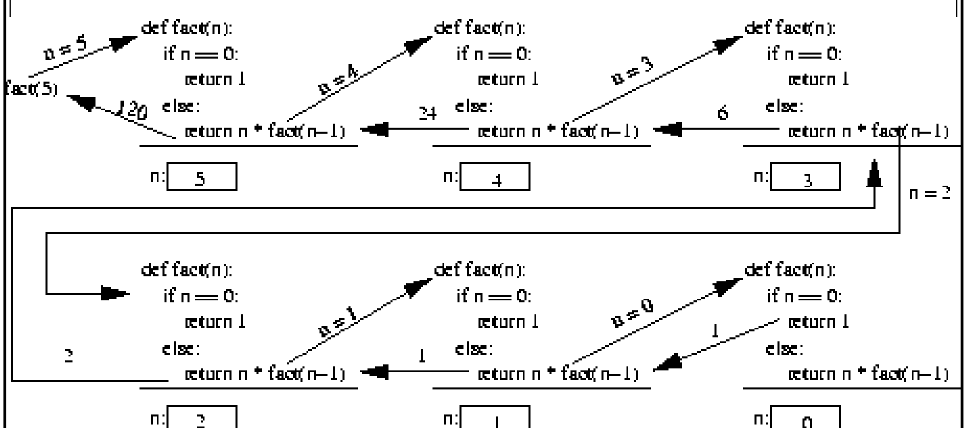
Recursive factorial

```
>>> fact(4)
24
>>> fact(10)
3628800
>>> fact(100)
9332621544394415268169923885626670049071596826438
  162146859296389521759999322991560894146397615651
  8286253697920827223758251185210916864000000000000
  00000000000000L
>>>
```

Remember: each call to a function starts that function anew, with its own copies of local variables and parameters.

Recursive Functions



Recursive fact(n)

```
def fact(n):
   if n==1:
      return 1
   else:
      return n*fact(n-1)
```

```
fact(5)
    fact(4)
        fact(3)
           fact(2)
                fact(1)
                return 1
           return 2*1=2
        return 3*2 = 6
    return 4*6 = 24
return 5*24 =120
```

Not a particularly useful program as the iterative version is more efficient and n! grows too quickly to use in computations

Recursive definitions: Fibonacci

- **Fibonacci Numbers**
  - **F(n) = F(n-1) + F(n-2) ,** F(0)=0 and F(1) = 1

    0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711 …
  - named after Leonardo of Pisa, but first described by an Indian mathematician (about 300BC)
  - Closed form solution exists

$$F(n) = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}},$$

where φ is the golden ratio; i.e., a root of $x^2 = x + 1$,