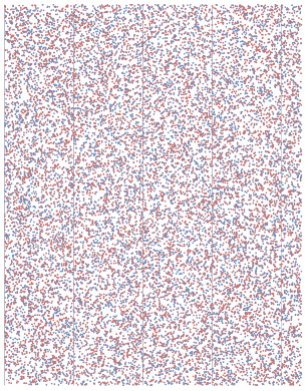
CS 190C
Science Education in Computational Thinking



RANDOMNESS:

- Floating-Point Numbers & Issues
- Using Randomness
- (Pseudo) Random Numbers
- Monte Carlo Methods
- Percolation

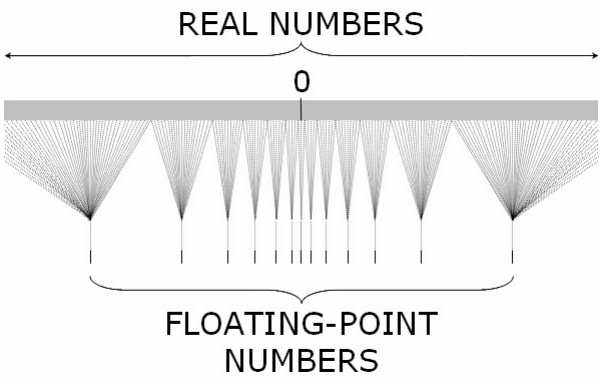
Hoffmann, 2008

Scientific Numbers

- 1.23664×10^4:
  - 10: base
  - 4: exponent
  - 1.23664: mantissa, precision 6 decimals
  - sign
- $1.23664 \times 10^4 + 1.56333 \times 10^7$
  - shift-align the mantissa 1.23664×10^4
  - do the subtraction $+1563.33000 \times 10^4$
  - round to 6-digit precision 1564.56664×10^4
  - result 1.56457×10^4

FP Arithmetic, IEEE 754

- Float: 1b sign + 8b exponent + 23b mantissa
- Double: 1b sign + 11b exponent + 52b mantissa
- 0.085 decimal =
  0 01111011 01011100001010001111011 binary
- Decimal examples with 3-digit mantissa:
  - $355 = 0.355 \times 10^3$
  - $12.77 = 0.128 \times 10^2$
- Rounding by magnitude instead of directed, but ALU can do directed rounding needed for interval computing
  - round -0.3555 **?** -0.356 or -0.355



Arithmetic Issues

- Precision = number of digits
  Accuracy = number of correct digits
  $\pi = 3.133333$ is precise to 7 decimals, but…

Clicker Question

Which of the following three is slowest?

A. Adding two 32bit integers
B. Fetching one 64bit double from RAM
C. Multiplying two 32bit floating point numbers

Arithmetic Issues

- Round-off error:
  $0.124 \times 0.351 = 0.043524$: 0.435×10^{-1}
  $\Delta = 0.24 \times 10^{-4}$
- Digit cancellation error:
  $0.127 - 0.124 = 0.003$: 0.300×10^{-2}
- Large summations are problematic, as are iterated computations in general…

Examples

- Square root by Newton, role of epsilon
- Digit cancellation near zero for $(1-\cos(x))/(x^2)$
- Events that happened due to arithmetic issues: http://www.cs.princeton.edu/introcs/91float/
  - Ariane 5 rocket 1996: arithmetic overflow
  - Patriot missile 1991: time increment $1/10^{th}$ sec
  - Intel FDIV hardware error 1997

pgm_0

Correct Graph?

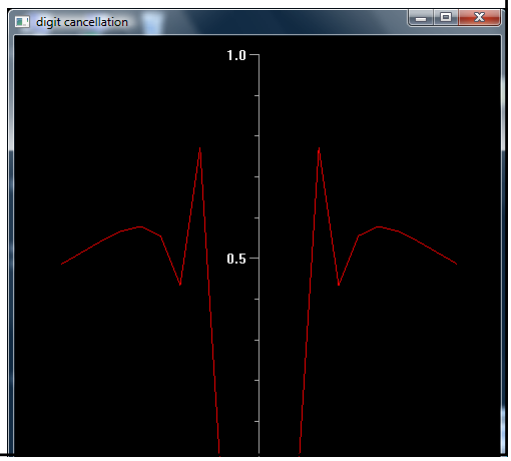
Near zero for $(1-\cos(x))/x^2$ is what?

$\cos(x) \approx 1 - x^2/2 + x^4/24 - x^6/6! \pm \ldots$

$1 - \cos(x) \approx x^2/2 - O(x^4)$

So, $(1 - \cos(x))/x^2 \approx ½ - O(x^2)$

We should see 0.5 at zero…



Which is not a Random Event?

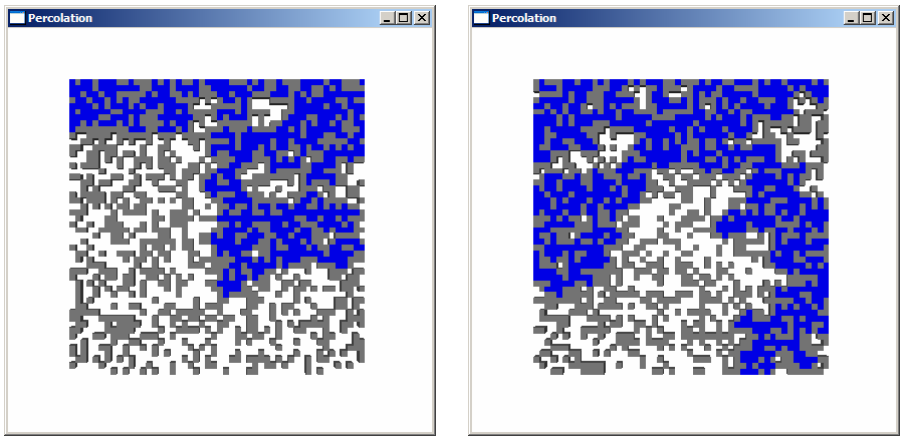
A. Color of first car crossing Stadium Ave after 12 noon

B. 101st digit in expansion of π is even

C. Fair coin toss

D. June 1, 2008, is a cloudy day

Percolation

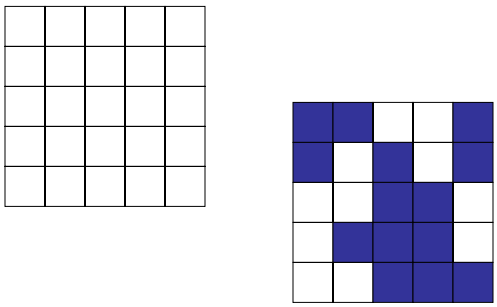


Probability for percolation given probability of generating the grid

Critical probability is 0.5

$pgm_{B,C}$

Percolation Grid is a Graph



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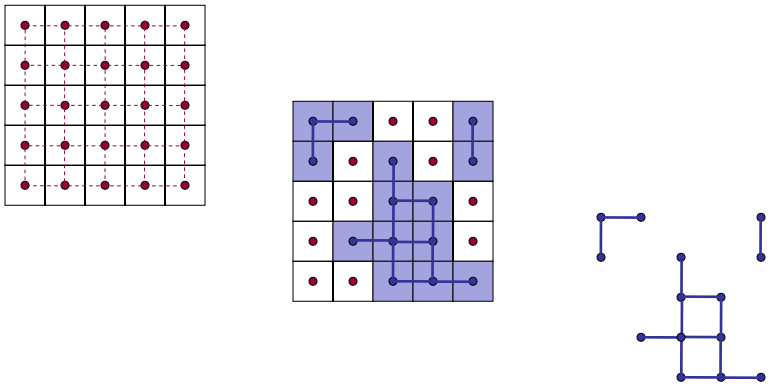


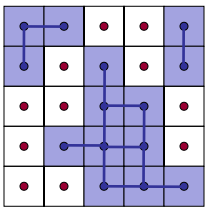
What Happens during Exploration?

- A recursive graph exploration:

```
visit(graph_node X):
    mark X as "visited"
    for each adjacent, not visited Y, where X → Y:
        visit(Y)
    return
```

- This is a depth-first search

DFS of Graph



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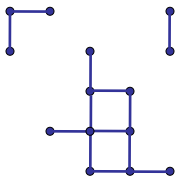

What Happens in Exploration?

- A recursive graph exploration, ending if we reach the other end:

```
visit(graph_node X):
   mark X as "visited"
   if X at bottom, return "percolates"
   for each adjacent not visited Y where X → Y:
      s = visit(Y)
      if s is "percolates": return s
   return "does not percolate"
```

- Driving by calling **visit(X)** with every **X** in top row