Topics for Today

- Introduction to graphs
- Basic operations and problems
- NetworkX
- Drawing graphs

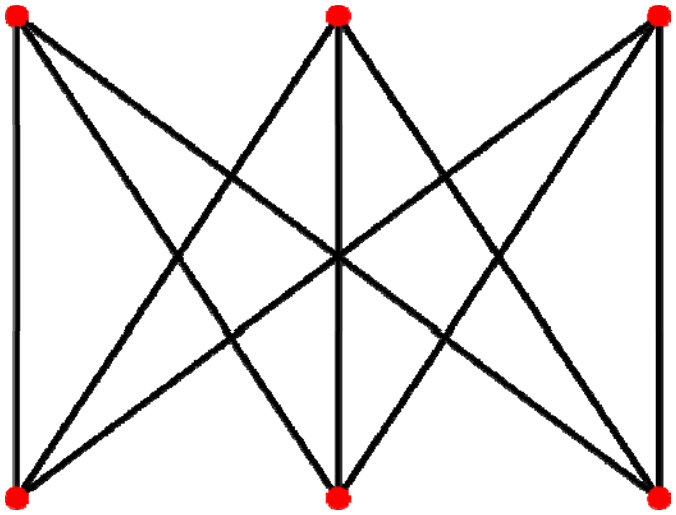
Reading

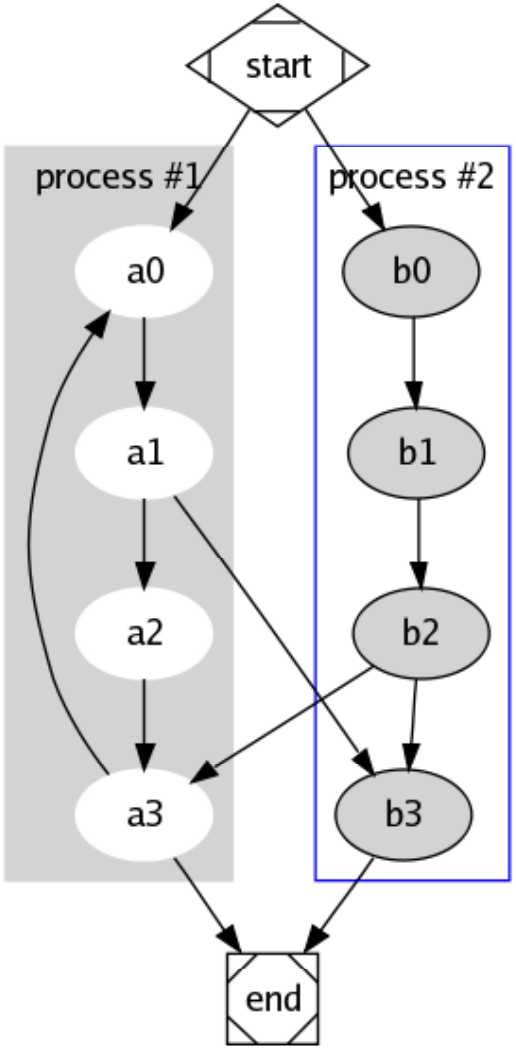
- https://networkx.lanl.gov/wiki/WikiStart - tutorial
- http://people.hofstra.edu/geotrans/eng/ch2en/meth2en/ch2m1en.html - gives more formal definitions
- http://en.wikipedia.org/wiki/Graph_theory - history and numerous links

Graphs

- Mathematical model for studying pair-wise connectivity among entities
- A graph is a discrete structure consisting of
  - nodes (also called vertices, sites) and
  - edges between nodes (also called links, connectors, arcs)
- Graphs are used to model problems in almost every discipline

Two graphs





	NODES	EDGES
Communication network	computers	fiber optic cable
Financial stocks	currency	transactions
Transportation	street intersections	highways
Protein interaction networks	proteins	protein-protein interactions
Internet	web pages	hyperlinks
Social networks	people	friendships
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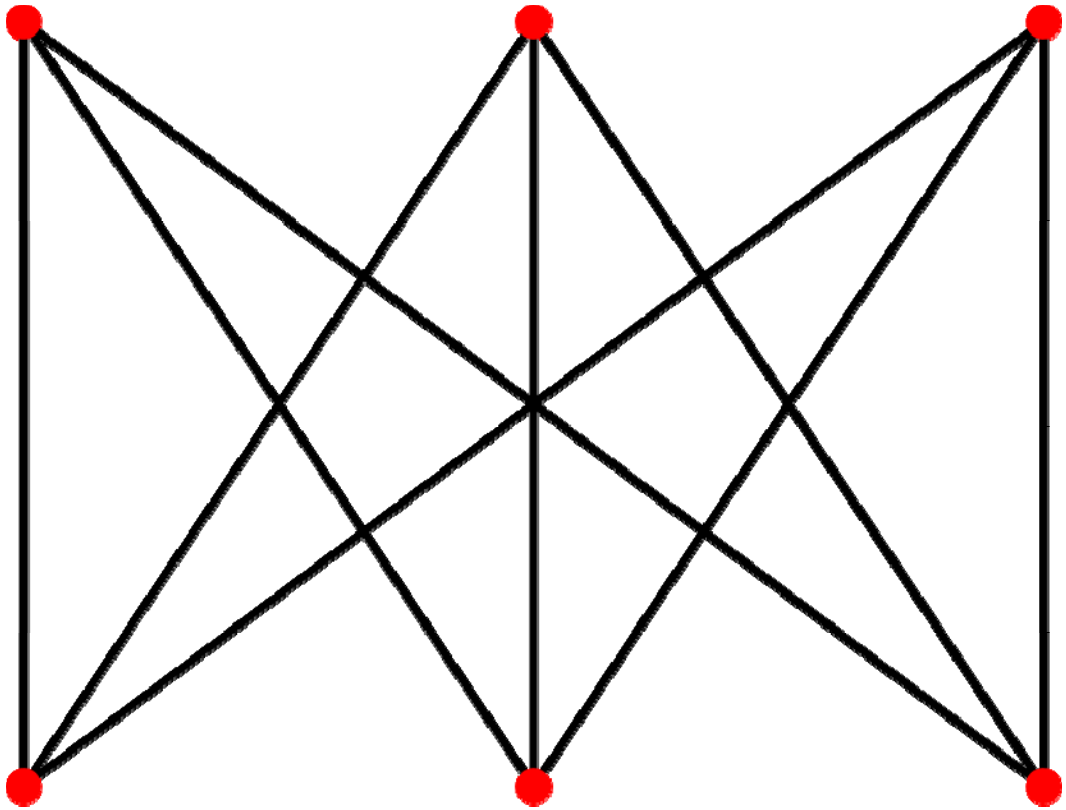
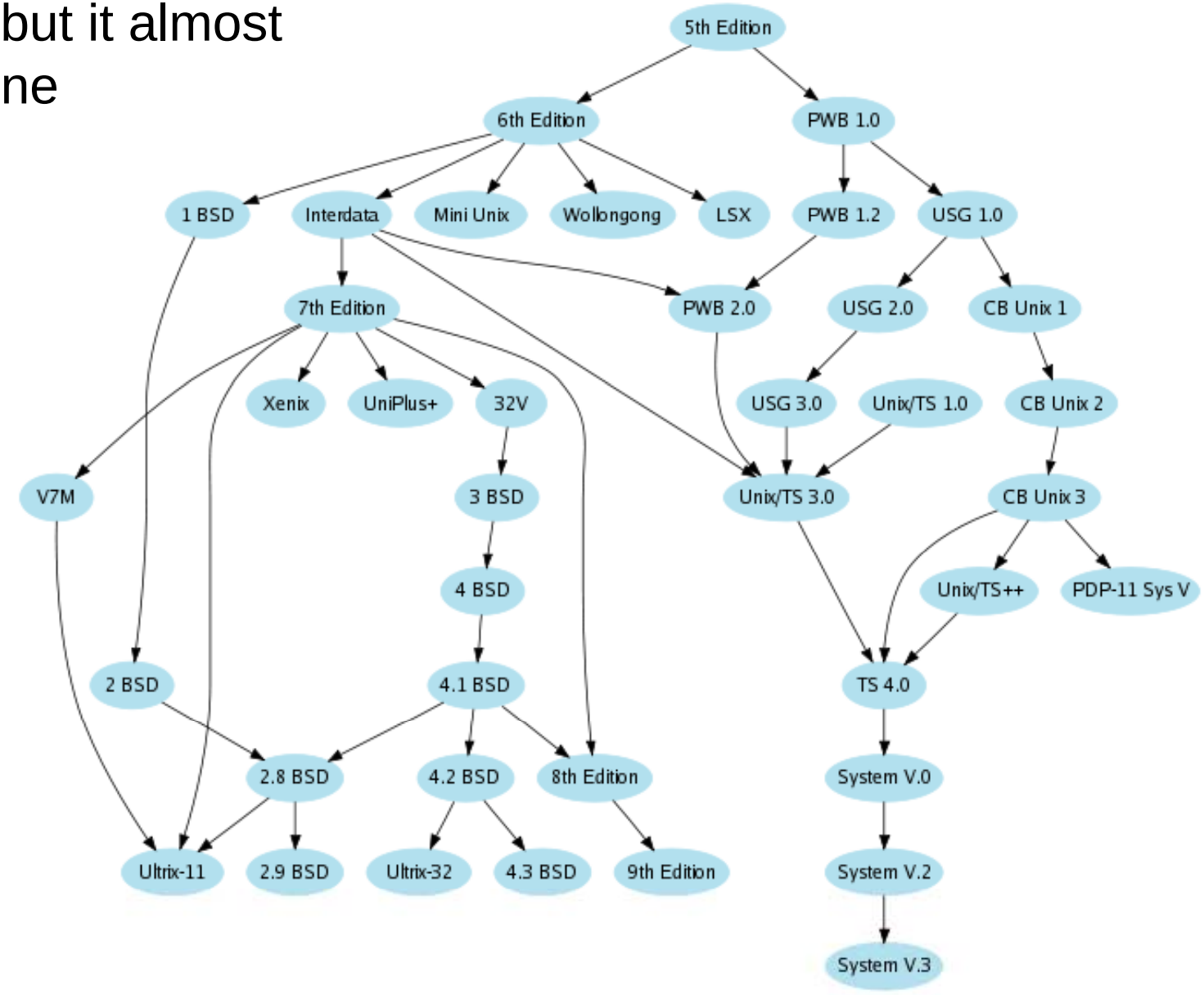
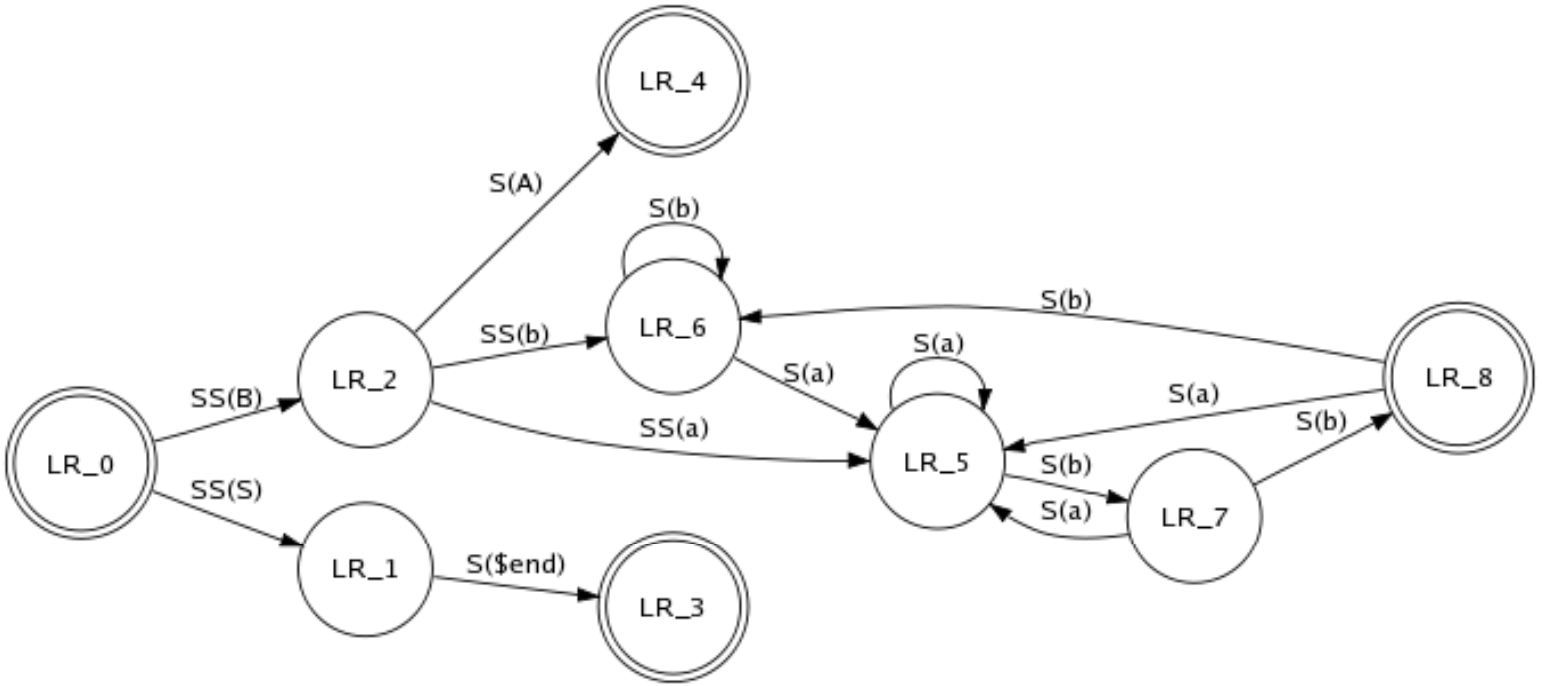
Graph consisting of 6 nodes and 9 edges
Edges have no directions,



Edges have directions
Not a tree, but it almost looks like one



Directed graph with cycles and self-loops
(edges have labels, nodes have labels)



Which of the following is not naturally modeled by a graph?

A. **Facebook users and their friends**
B. **Spread of a disease**
C. **Weather prediction**
D. **Course requirements and prerequisites**
E. **Electric power distribution system**

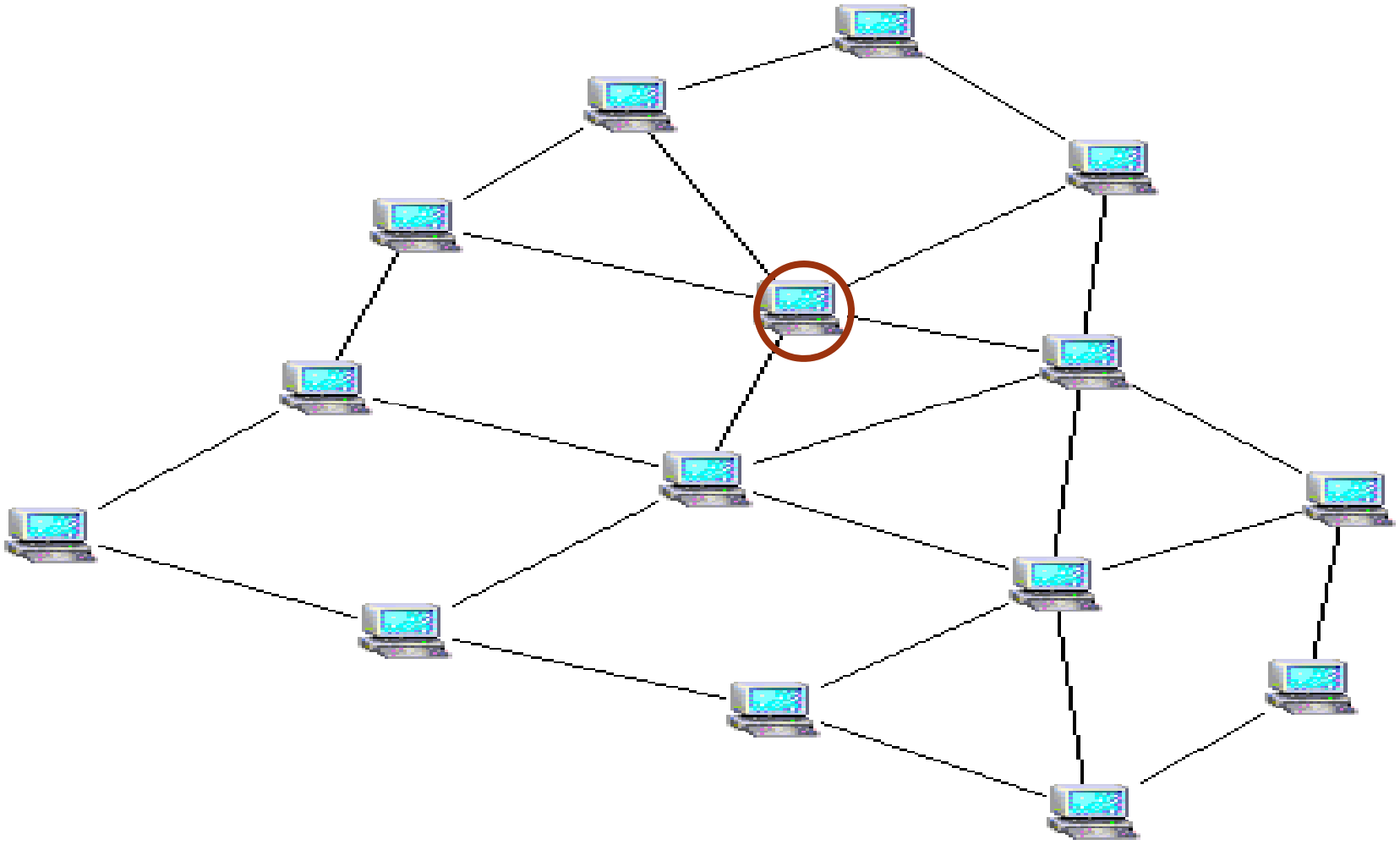
Some terminology

- Graphs have a finite number of nodes, but they can grow and shrink
- Undirected and directed graphs
- Weights on edges: weighted and unweighted graphs
- Nodes generally have names
- At most one edge between a pair of nodes: simple graph (otherwise a multi-graph)
- self-loops are often excluded as possible edges

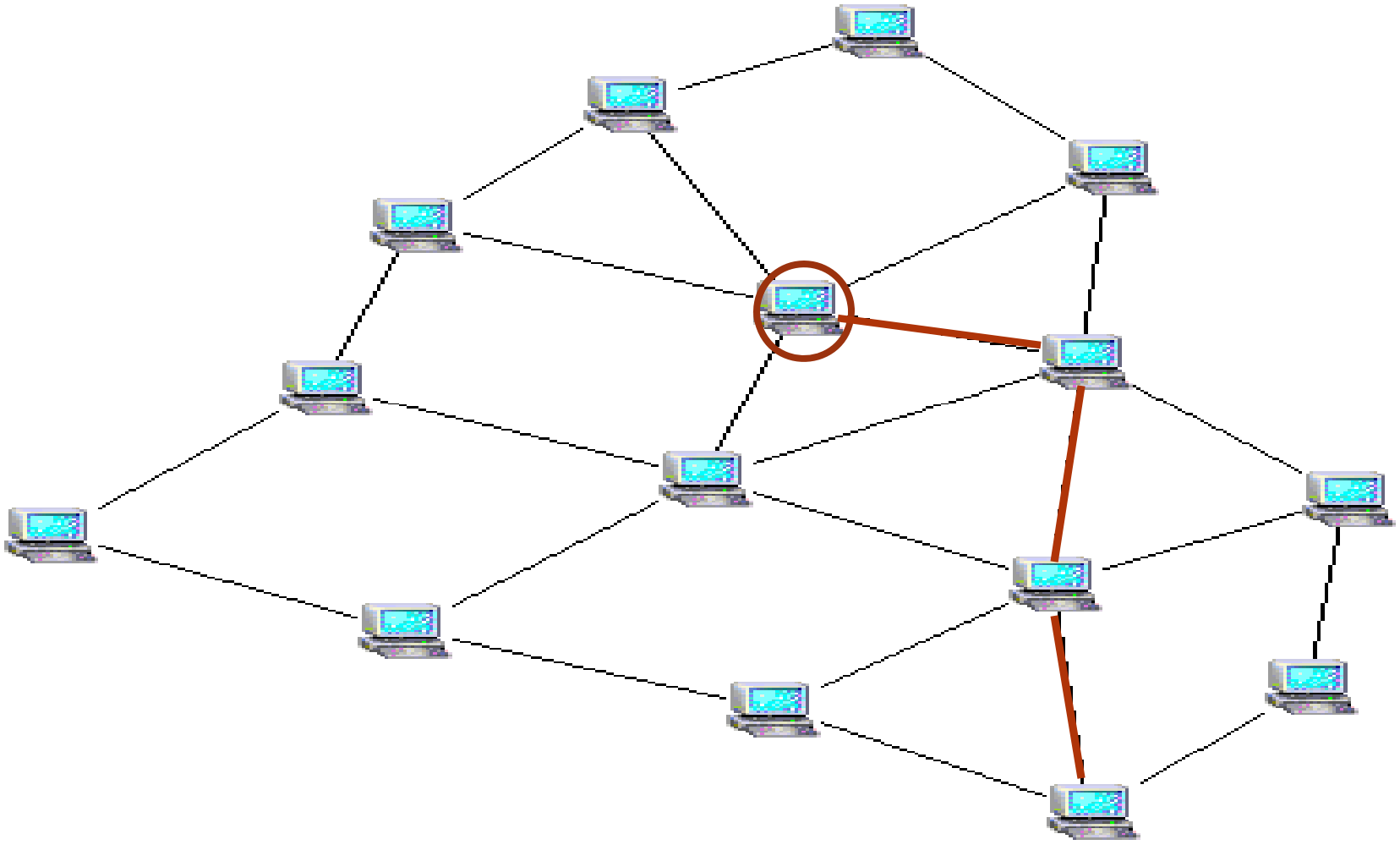
More terminology

- A tree is a special type of graph
- Undirected graph
  - a node is "adjacent" to other nodes
  - these nodes are also called its neighbors
- Directed graph
  - a node has edges going out and edges coming in (in-degree and out-degree of a node)
  - adjacency is often the number of edges leaving the node
- Path between two nodes – sequence of edges connecting them

Undirected, simple graph

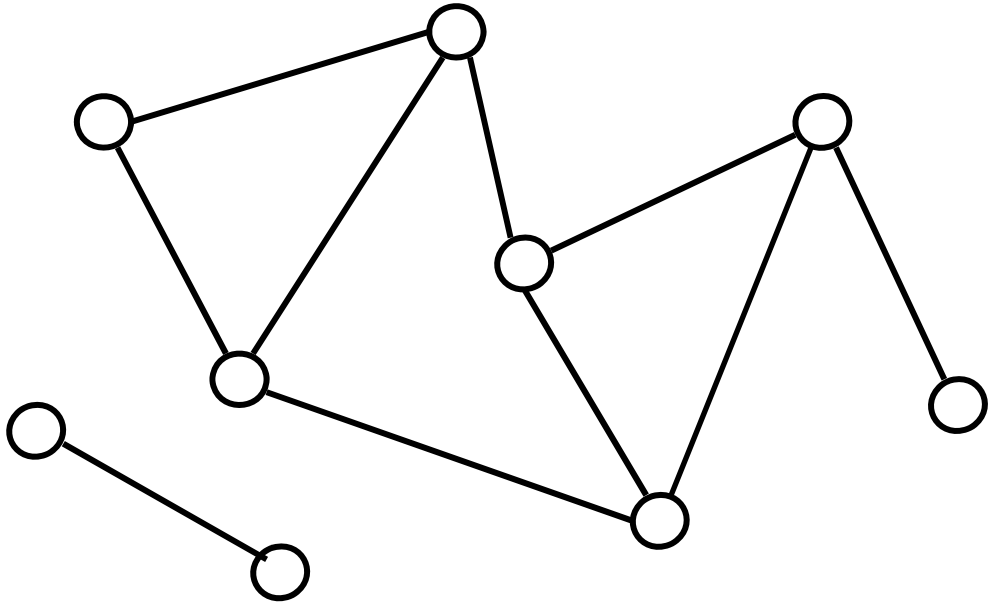


Undirected, simple graph



How many nodes have degree 3?

A. 3
B. 4
C. 5
D. 6



Basic functionality we need

- Create a graph
  - read edges from a file
  - add and delete edges and nodes
  - Perform the union graphs
  - Pull out subgraphs
- Iterate through all the nodes of a graph
- For a given node u, iterate through all its the adjacent nodes
- Print /visualize a graph

Representing Graphs

- We will not discuss the internal representation of graphs
- NetworkX will do it for you

Otherwise …

- As a 2-dimensional matrix (adjacency matrix)
  - A[a][b] =1 if there exists an edge from node a to b (can also store the edge weight)
- As a list of lists (adjacency lists)
  - i-th list contains all the nodes node i is adjacent to

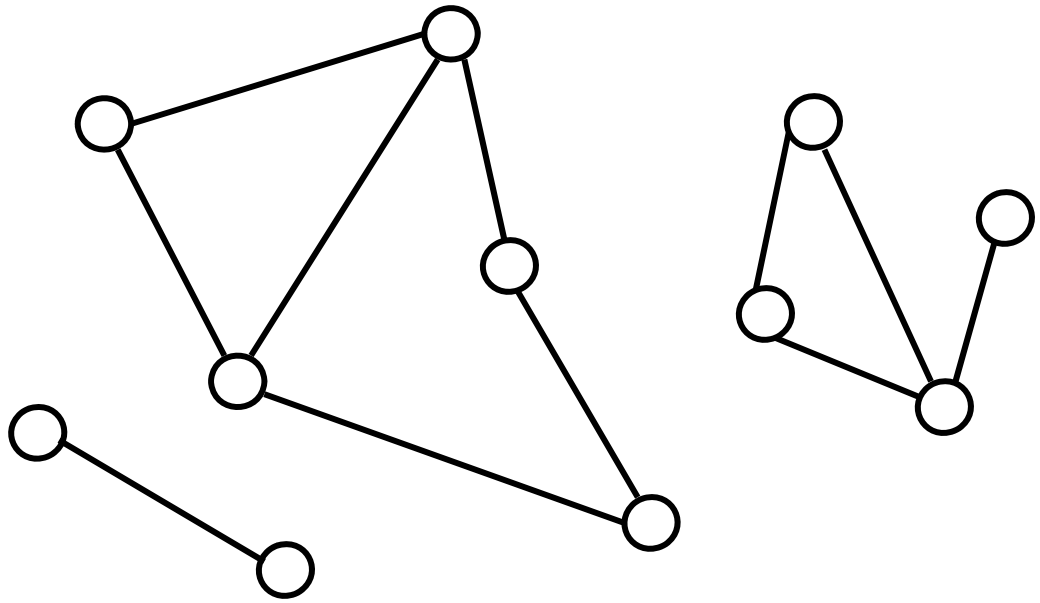
Problems on undirected, simple graphs

- Is graph G connected?
- If it is not connected, identify its connected components
- Is the graph a tree?
- Create a subgraph from a graph

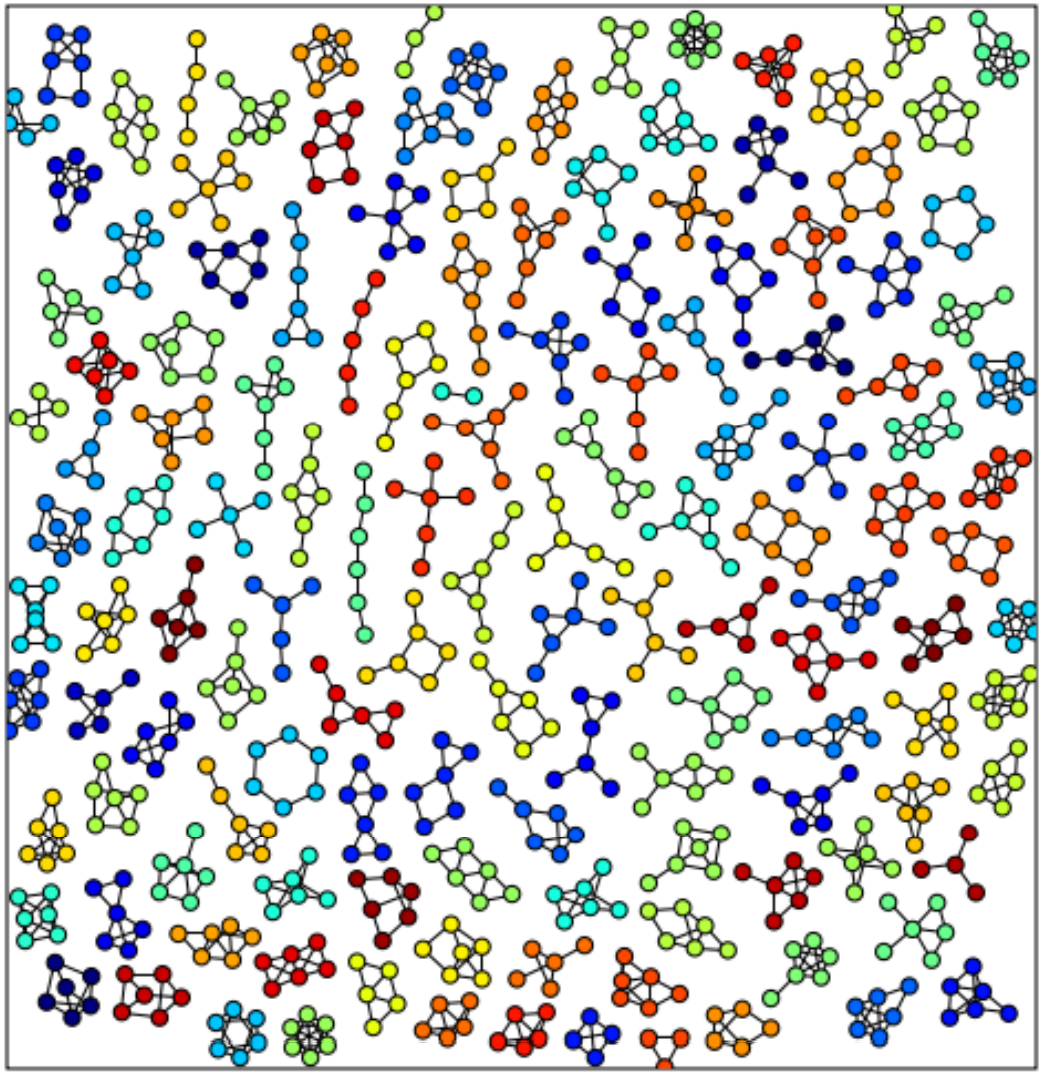
- Explore all nodes in a specified order (depth-first-search, breadth first search)
- Find paths of shortest length between nodes

What is the minimum number of edges that have to be added to make the graph connected?

A. 1 edge
B. 2 edges
C. 3 edges
D. 4 edges



Undirected, simple with colored connected components

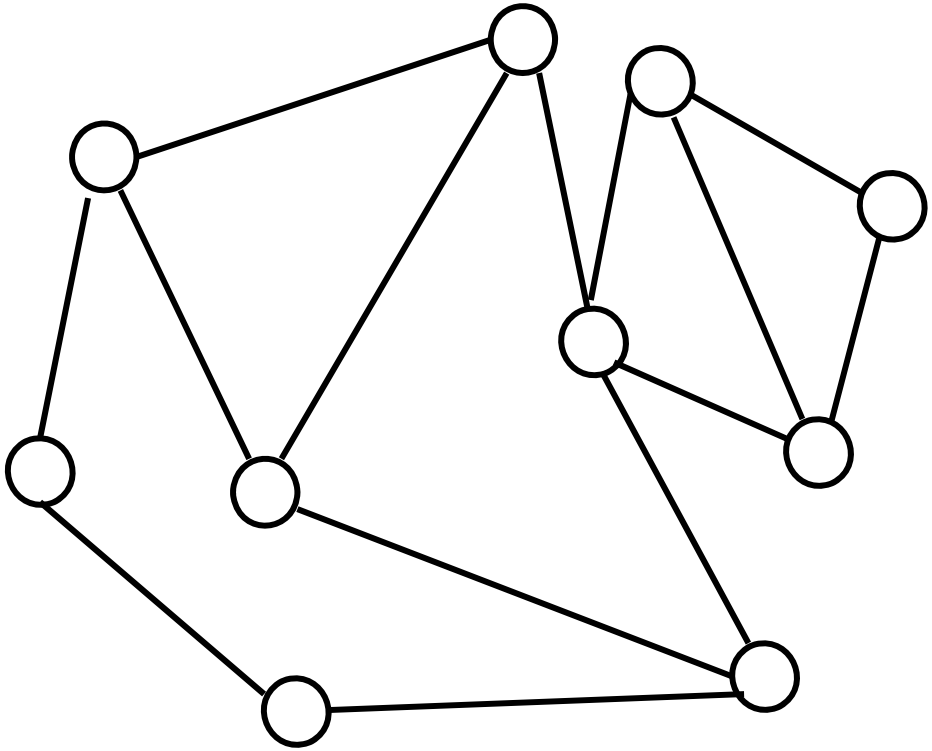


What is the minimum number of nodes that have to be removed to create at least two connected components of size >1?

A. 1

B. 2

C. 3



NetworkX

https://networkx.lanl.gov/wiki/WikiStart

Read Tutorial and Quick Reference sections

- Easy to get started and intuitive to use
- Poor visualization of graphs; complete reference material overwhelming

```
from networkx import *
import pylab as P
```

NetworkX Basics

undirected1.py
undirected2.py

```
G = Graph()

G.add_edges_from([tuple(s.split()) for s in
                                open('graph1.txt')])

print G.edges()
print G.nodes()
CC = component.connected_components(G)

draw(G)
P.show()
```

Drawing graphs generated by NetworkX

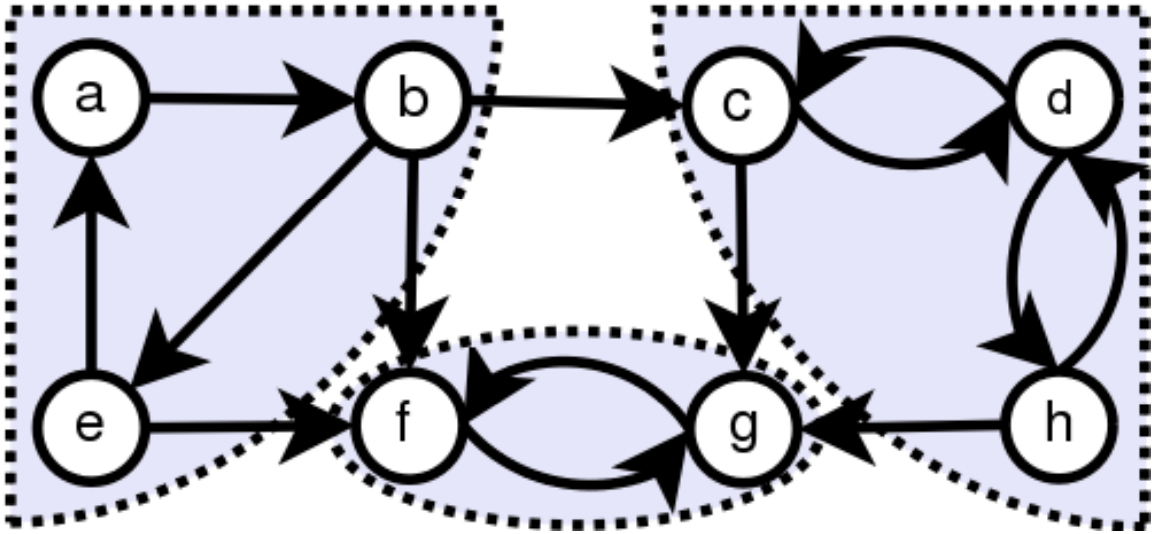
```
G_com = complete_graph(5)
G_star = star_graph(n)
G_lad = circular_ladder_graph(n)
G_hyp = hypercube_graph(4)
G_ran1 = erdos_renyi_graph(2*n, p, seed=None)

P.figure(1)
draw(G_star)

P.figure(2)
draw(G_lad)

P.figure(3)
draw_spectral(G_ran1)

P.figure(4)
draw_circular(G_com)
```

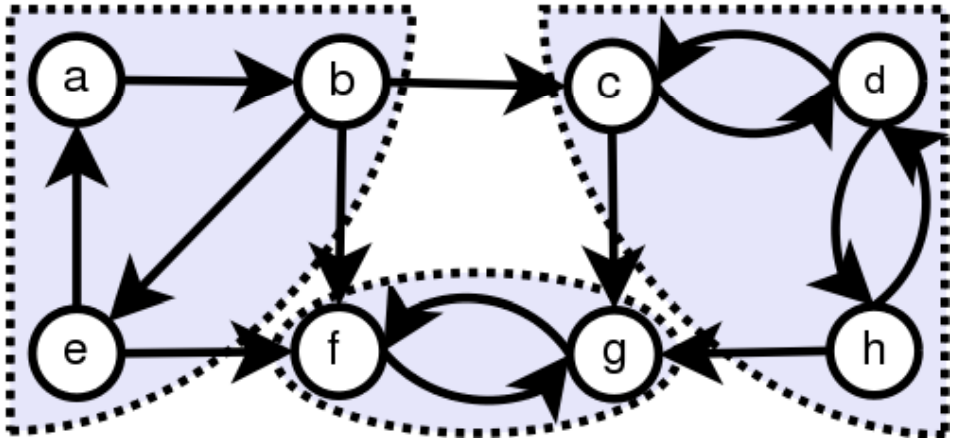
Some graph operations

```
G1 = complete_graph(5)
G2 = star_graph(5)
G3 = ladder_graph(4)
H = Graph()
H = union(G1,G2, rename=('G-','H-'))
print "len(H): ", len(H)

H = union (H,G3)
HN = convert_node_labels_to_integers(H)

list_ccHN = connected_components(HN)
print list_ccHN

sub_HN = subgraph(HN, list_ccHN[0])
```

Directed, simple graphs

- What does "connected" mean?
- A directed graph is *strongly connected* if there is a (directed) path between any two nodes
- If it is not strongly connected, identify its strongly connected components





Directed graph has 8 nodes

3 strongly connected components
[[a,b,e], [f, g], [c,d, h]]

Connected components represented as a list of list

Operations on directed graphs

```
G = DiGraph()
G.add_edges_from([tuple(s.split()) for s in open('EBI.txt')])

print 'Number of nodes:', G.order()
print

C1 = component.strongly_connected_components(G)

print 'Number of strongly connected components:', len(C1)
print 'Strongly connected component sizes > 3: ‘
for t in C1:
        if len(t) > 3:
            print "a strongly connected component of size: ", len(t)
print
```